LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **NOVEMBER 2023**

UST 3502 – MATRIX AND LINEAR ALGEBRA

Date: 04-11-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

	SECTION A - K1 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
1.	**Define the Following**
a)	Trace of the Matrix.
b)	Inverse of a matrix.
c)	Orthogonal Transformation.
d)	Power of a matrix.
e)	Signature of the matrix.
2.	**True or False**
a)	Rank of the matrix A_{3x4} can be 4.
b)	The inverse of the matrix A_{3x3} always exists.
c)	The equation AX = b is called homogeneous if b= 0.
d)	Cayley – Hamilton theorem satisfied any non-square matrices.
e)	The number of –ve square terms in the Q.F is called the Index of Q.F.
	SECTION A - K2 (CO1)
	Answer ALL the Questions **(10 x 1 = 10)**
3.	**Fill in the blanks**
a)	Inter changing of any two rows or column in the matrix change the ___________________.
b)	Cramer's rule is applicable only for _____________ Matrices.
c)	The set { u1,u2,……..uk} is linearly dependent iff ______________.
d)	The Eigen value of A is 3,-4 and 0 . Then, the Eigen Value of A^3 is ____________.
e)	All Characteristic Values are positive , the Q.F is called _______.
4.	**Answer the following**
a)	Find the cofactors of $A = \begin{bmatrix} 3 & -2 \\ 5 & 4 \end{bmatrix}$.
b)	What is the inverse of the matrix $A = \begin{bmatrix} 1 & 7 \\ -3 & 4 \end{bmatrix}$?
c)	What is mean by Basis?
d)	Find Inverse of A by using Cayley- Hamilton theorem from the equation $A^2+3A+5I = 0$,Where $A = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$
e)	Write the nature of the Quadratic Form: $X_1^2 +2 X_2^2 - X_3^2$.
	SECTION B - K3 (CO2)
Answer any TWO of the following	**(2 x 10 = 20)**
5.	Write all properties of Determinants.
6.	Solve the equations by using Cramer's Rule: $2x+4y+z = 5$; $x+y+z= 6$; $2x +3y +z = 6$.

7.	Determine whether the set S = $\left\{\begin{bmatrix}1\\2\\1\end{bmatrix},\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}1\\4\\1\end{bmatrix},\begin{bmatrix}1\\2\\3\end{bmatrix}\right\}$ is linearly dependent or linearly independent.
8.	Two of the Eigen values of $\begin{bmatrix}6&-2&2\\-2&3&-1\\2&-1&3\end{bmatrix}$ are 2 and 8. Find the 3rd Eigen value and also find its Eigen vector and its determinant.

SECTION C – K4 (CO3)

Answer any TWO of the following (2 x 10 = 20)

9.	If A = $\frac{1}{\sqrt{2}}\begin{bmatrix}1&-1\\1&1\end{bmatrix}$, Show that A is Orthogonal.
10.	Verify Cayley- Hamilton theorem then find A^4. When A = $\begin{bmatrix}2&-1&2\\-1&2&-1\\1&-1&2\end{bmatrix}$
11.	Show that 3 and -2 are Eigen values of the linear operator T on R^2 define by T $\left(\begin{bmatrix}x1\\x2\end{bmatrix}\right)=\begin{bmatrix}-2x1\\-3x1+x2\end{bmatrix}$ and find bases for the corresponding eigen spaces.
12.	Find the Eigen vectors of A = $\begin{bmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{bmatrix}$.

SECTION D – K5 (CO4)

Answer any ONE of the following (1 x 20 = 20)

13.	(i) Prove that $\begin{vmatrix}x&x^2&yz\\y&y^2&xz\\z&z^2&xy\end{vmatrix}=(x-y)(y-z)(z-x)(xy+yz+zx)$ (ii) If A = $\begin{bmatrix}1&2&-3\\5&0&2\\1&-1&1\end{bmatrix},B=\begin{bmatrix}4&1&2\\0&3&2\\1&-2&3\end{bmatrix},C=\begin{bmatrix}3&-1&2\\4&2&5\\2&0&3\end{bmatrix}$ The compute (A+B) and (B-C) and also verify that A + (B-C) = (A +B) - C
14.	Solve the following system of linear equations: X1+2X2-X3+2x4+x5 = 2 -x1-2x2+x3+2x4+3x5=6 2x1+4x2-3x3+2x4 =3 -3x1-6x2+2x3+3x5 =9

SECTION E – K6 (CO5)

Answer any ONE of the following (1 x 20 = 20)

15.	(i)Calculate $[T]_B$, If T and B are the linear operator and the basis B { b1,b2,b3}. Where b1 = $\begin{bmatrix}1\\1\\1\end{bmatrix},b2=\begin{bmatrix}1\\2\\3\end{bmatrix},b3=\begin{bmatrix}2\\1\\1\end{bmatrix}$ and $T\left(\begin{bmatrix}X1\\X2\\X3\end{bmatrix}\right)=\begin{bmatrix}3x1+x3\\x1+x2\\-x1-x2+3x3\end{bmatrix}$

	(ii) Verify Cayley – Hamilton for the matrix $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$.
16.	Reduce the Quadratic form below to its normal form by an orthogonal reduction $3X_1^2+2X_2^2+3X_3^2-2X_1X_2-2X_2X_3$. Using the result and find A^4.

&&&&&&&&&&